It was rudely pointed out to me that I have incorrectly assumed
that all numbers are in this beautiful "tree" structure. This has to be proved.

Collatz Tree V1.0 - July 31, 2024 - Dave Cromley

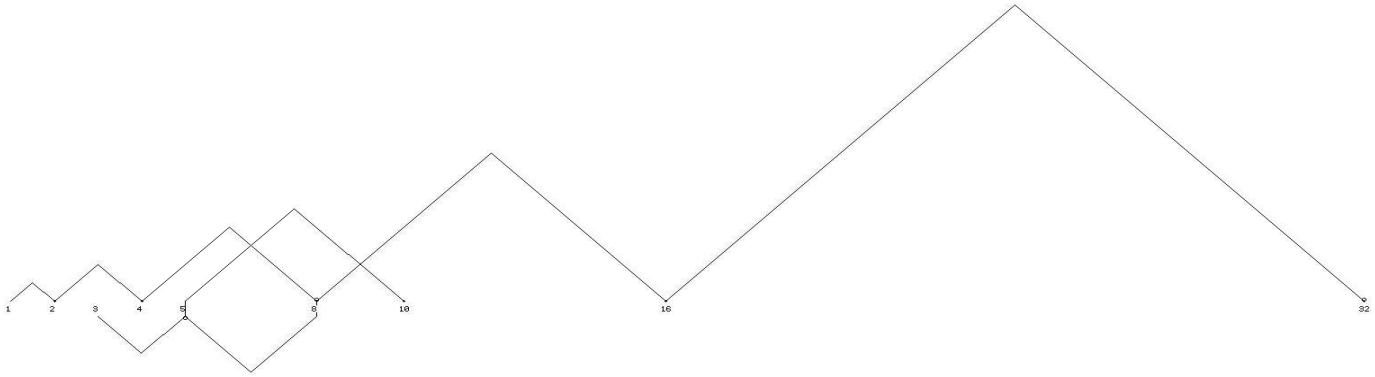
This paper uses the terminology from the excellent wikipedia article on the Collatz conjecture.

Abstract: The Collatz tree can be formed naturally, resulting in a good looking tree, but the numbers seem chaotic. By reforming the tree, the tree becomes weird, but the numbers look nice. And it becomes clear that the hailstone algorithm and its inverse can be used to traverse down and up the tree, from the root to all integers and beyond, and back to the root (the number 1).

This paper relies on a tree structure that represents the shortcut Collatz tree that is defined by

- 1 is the root
- the branch from node n to the next node is given by
  - All nodes branch from n to 2n
  - If n = 2 mod 3 then the node also branches to (2n-1)/3

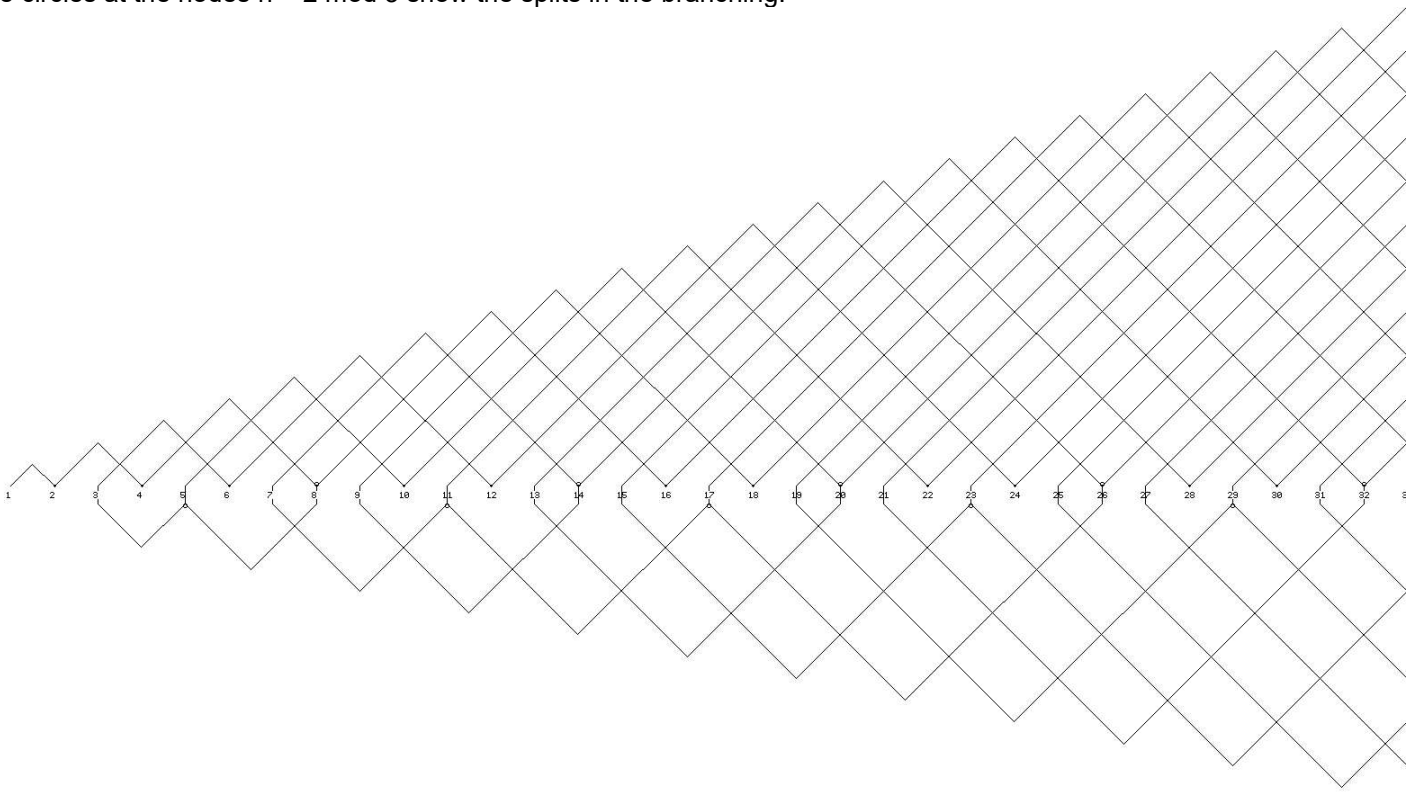
Fig. 1) The beginnings of the Collatz tree:



Nodes of distance 1 to 5 from the root:

dist=1: 2 dist=2: 4 dist=3: 8 dist=4: 5 and 16 dist=5: 3, 10 and 32

Fig. 2) The left part of the Collatz tree with nodes at a distance up to 71 from the root:
The circles at the nodes n = 2 mod 3 show the splits in the branching.



The tree grows up to the right above the nodes (integers), and grows down to the left below the nodes. I don't think this tree form appears in nature.

This tree structure makes it clear how to traverse from any node away from the root or towards the root, and even to the root (1). Traversing from the root to a given n is difficult. Traversing from a given n to the root (1) is done by the determinate shortcut hailstone algorithm, one branch at a step.

Choose any integer n. We will assume it is in the tree at a distance k from the root. We will traverse to the next node closer to the root by the shortcut hailstone rules:

- If n is even then n' = n/2.
- If n is odd then n' = (3n+1)/2.

Referring to the Collatz tree, n' is closer to the root by 1 branch (k-1).
This can be continued until the root (1)(k=0) is reached. Which proves the Collatz conjecture.

I plan to port some programs that show the growth of the Collatz tree from BASIC to javascript, and post them on my website.

Of course, I realize that I may be kidding myself and be a victim of wishful thinking.

The link to this paper: http://dbarc.net/yr2024/collatz1.0.pdf

Dave Cromley
dave71@dbarc.net
(307) 286-7513